

# Energy Storage for IP over WDM Networks with Renewable Energy

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**Abstract**—We consider the dynamic selection of lightpaths and optical fiber topology to minimize energy consumption in an optical transport network with renewable energy. We assess the performance and energy consumption profiles of networks that leverage energy storage in conjunction with renewable energy. We demonstrate that the use of energy storage decouples the network planning and traffic engineering problem from the renewable energy availability. This reduces path lengths, lightpath reconfigurations and the number of router ports required.

## I. INTRODUCTION

The rapidly growing use of mobile devices and the increasing availability of high speed access is fuelling the growth of traffic on the Internet. One report measured that capacity will need to increase by a factor of 12X by the year 2022 [1]. To support this growth, the Information and Communications Technology (ICT) industry is expanding networks and consequently increasing energy consumption and carbon emissions. A report by The Climate Group predicted the environmental footprint of the communications infrastructure will grow at 5% annually doubling by 2020 [2]. In an effort to reduce operating expenses and environmental footprint there has been a push to green the communications infrastructure. One approach is the integration of renewable energy technology to power the infrastructure since renewable energy has low operating expense and offsets greenhouse gas emissions produced by other energy sources. A concern with that is the energy must be consumed as it is produced in order to justify the capital expense. To do this in a backbone network requires traffic engineering to choose path assignments according to renewable energy availability.

In this work we demonstrate how utilizing energy storage in conjunction with renewable energy systems can decouple the renewable energy availability from the network planning and traffic engineering problem. We consider an IP over WDM network with optical bypass and wavelength conversion capability. We examine three specific scenarios for comparison. In the first case we consider a network with no renewable energy representing a baseline case. In the second case we consider the availability of solar renewable energy at a subset of sites in the network. In the third case we assume that energy storage is available at the sites with access to solar energy. We compare the cases on the basis of environmental impact, cost and performance. Using the NSFNET network as a reference we show that although solar energy without energy storage reduces the total non-renewable energy consumption, there are several negative impacts:

- Network path lengths increase up to 40-50%

- Lightpath reconfigurations increase by up to 4X
- Number of router ports required increases up to 45-50%

We demonstrate that energy storage further reduces the consumption of non-renewable energy without the negative impacts observed in the case with solar energy without energy storage. We also study the solar panel sizing problem and show how a router's daily energy requirements can be met solely by renewable energy. To the best of our knowledge this is the first work examining the use of energy storage systems in conjunction with renewable energy in IP over WDM networks.

## II. RELATED WORK

Recognizing the increasing operating expenses and carbon footprint of service provider networks, recent research has focused on integrating renewable energy into backbone networks. In [3] an LP optimization and heuristic algorithm is presented for minimizing non-renewable energy consumption in a hybrid-power IP over WDM network with time-varying traffic. The proposed heuristic algorithm controls delay by selecting paths which are both short in length and have renewable energy available. In [4] a joint optimization of power, electricity cost and delay in IP over WDM networks is developed. To maintain linearity the approach measures delay in terms of lightpath hops. The objective of the minimization is a weighted sum of power, electricity and lightpath hops with predetermined tradeoff coefficients. The work [5] established the lower bound for energy consumption and greenhouse gas emissions in a network with renewable and legacy power sources with static routing and wavelength assignment using an ILP optimization. It found that a significant reduction in non-renewable energy consumption could be achieved with 25% of sites equipped with green energy sources.

In order for a network to consume renewable energy as it is being produced, it must be constantly re-configured. Fibers must be activated and deactivated and the virtual topology must be reconfigured and re-routed over the physical topology. From a service provider standpoint topology reconfigurations are costly and introduce potential delays and traffic disruptions. In [6] heuristic algorithms for power-aware virtual topology design problem are presented. The algorithms aim to balance energy consumption and the amount of traffic disrupted when the network is reconfigured. The work [7] measures the various impacts of lightpath reconfiguration including added delay, QoS degradation, and energy consumption. It also presents a heuristic algorithm for determining the optimal ordering of reconfigurations yielding minimum impact.

## III. PROBLEM FORMULATION

We consider the energy optimization of an IP over WDM network with optical bypass and wavelength conversion. Optimization is performed for a time period in which the traffic demand and renewable energy output remain fixed, e.g. an hour. As in [8] network power consumption is composed of *router ports* required to aggregate traffic from access networks as well as at the end points of each lightpath, *optical transponders* at the end points of each wavelength connecting two optical nodes, and *optical amplifiers* along each active fiber link. For each scenario the optimization performs both network planning and traffic engineering by determining:

- The number of fibers to activate on each link
- The virtual lightpath topology
- The assignment of lightpaths to fibers
- The assignment of traffic to lightpaths

We consider three cases for comparison. In the first case ('No Solar') we assume there is no renewable energy available in the network. In the second case ('Solar') we assume the presence of solar renewable energy at a subset of the router sites. In the third case ('Solar with Storage') we introduce energy storage at the sites with access to solar renewable energy. All three cases will be based on the following set of parameters and variables where the indices  $i$  and  $j$  denote nodes in the virtual layer,  $m$  and  $n$  nodes in the physical layer and  $s$  and  $d$  traffic source and destination nodes:

*Parameters:*

- $r^{sd}$ : Traffic demand between nodes  $s$  and  $d$  in Gbps.
- $g_m$ : Renewable energy availability indicator. 1 if node  $m$  has access to renewable energy and 0 otherwise.
- $p_m$ : renewable energy power output at node  $m$  in Watts
- $B$ : The bandwidth of a wavelength in Gbps.
- $W$ : The number of wavelengths on a single fiber.
- $N$ : The number of nodes in the network.
- $N_m$ : The set of nodes neighboring node  $m$ .
- $\Delta_i$ : The number of router ports required to aggregate traffic entering from access networks at node  $i$ .
- $\nabla^i$ : The number of router ports available at node  $i$ .
- $E_r, E_t, E_e$ : Average power consumed by a router port, optical transponder and optical amplifier respectively.
- $S$ : Distance required between EDFA amplifiers in km.
- $D_{mn}$ : The length of link  $(m, n)$  in km.
- $F_{mn}$ : Number of fibers available on physical link  $(m, n)$ .
- $\beta_m^{in}, \beta_m^{out}$ : The amount of energy stored at node  $m$  in Joules at the beginning and end of the time period respectively.
- $\beta^{max}$ : Maximum energy storage capacity in Joules.

*Variables:*

- $C_{ij}$ : Number of lightpaths connecting nodes  $i$  and  $j$ .
- $r_{ij}^{sd}$ : Amount of traffic in Gb/s on lightpath  $(i, j)$  for source destination pair  $(s, d)$ .
- $w_{mn}^{ij}$ : Number of wavelengths on physical link  $(m, n)$  for lightpaths connecting logical nodes  $(i, j)$
- $f_{mn}$ : Number of activated fibers on physical link  $(m, n)$ .

## A. Case 1: No Solar

In the first case, the total power consumption for a fixed time interval and given traffic demand is minimized and is based on the work presented in [9]. The following objective function minimizes the power consumed by router ports, optical transponders and optical amplifiers:

$$\begin{aligned} \text{minimize} \quad & E_r \cdot \left[ \sum_{i \in N} \sum_{j \in N, i \neq j} C_{ij} + \sum_{i \in N} \Delta_i \right] \\ & + E_t \cdot \left[ \sum_{m \in N} \sum_{n \in N, m \neq n} w_{mn} \right] \\ & + E_e \cdot \left[ \sum_{m \in N} \sum_{n \in N, m \neq n} f_{mn} \cdot \left\lceil \frac{D_{mn}}{S} \right\rceil \right] \end{aligned} \quad (1)$$

where

$$w_{mn} = \sum_{i \in N} \sum_{j \in N, i \neq j} w_{mn}^{ij} \quad (2)$$

is the total used wavelengths used on physical link  $(m, n)$ .

**subject to** constraints (3) - (9) described next. The following is the flow conservation constraint in the virtual layer:

$$\sum_{j \in N, i \neq j} r_{ij}^{sd} - \sum_{j \in N, i \neq j} r_{ji}^{sd} = \begin{cases} r^{sd}, & i = s. \\ -r^{sd}, & i = d. \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

$$\forall s, d, i, \in N, s \neq d.$$

The amount of traffic carried on a link in the virtual layer is constrained by the number of lightpaths connecting the logical nodes and the capacity of a lightpath:

$$\sum_{s \in N} \sum_{d \in N, s \neq d} r_{ij}^{sd} \leq C_{ij} \cdot B \quad \forall i, j \in N, i \neq j. \quad (4)$$

The number of router ports used at a node is constrained and is determined by the number of lightpaths incident to the node and the number of ports required for aggregating incoming access network traffic:

$$\sum_{j \in N, i \neq j} C_{ij} + \Delta_i \leq \nabla^i \quad \forall i \in N. \quad (5)$$

$$\sum_{i \in N, i \neq j} C_{ij} + \Delta_j \leq \nabla^j \quad \forall j \in N. \quad (6)$$

Flow conservation is also required in the physical layer:

$$\sum_{n \in N_m} w_{mn}^{ij} - \sum_{n \in N_m} w_{nm}^{ij} = \begin{cases} C_{ij}, & i = m. \\ -C_{ij}, & j = m. \\ 0, & \text{otherwise} \end{cases} \quad (7)$$

$$\forall i, j, m \in N, i \neq j.$$

The number of wavelengths available on a physical link is constrained by the number of active fibers and the wavelength capacity of an individual fiber:

$$\sum_{i \in N} \sum_{j \in N, i \neq j} w_{mn}^{ij} \leq W \cdot f_{mn} \quad \forall m \in N, n \in N_m. \quad (8)$$

Finally the number of active fibers is constrained by the availability of physical fibers on a link:

$$f_{mn} \leq F_{mn} \quad \forall m \in N, n \in N_m. \quad (9)$$

### B. Case 2: Solar

In the second case ('Solar') we consider the scenario where a subset of the nodes have access to solar energy. Systems that deploy renewable energy attempt to consume as much energy as is produced in order to minimize the per kWh cost. In case 2 the objective is to minimize the consumption of power from non-renewable sources in an effort to make the best use of the solar energy resource. There are several factors which contribute to the non-renewable energy. We begin with the router port power consumption at sites without renewable:

$$E_r \cdot \left[ \sum_{i \in N} \sum_{j \in N, j \neq i} C_{ij} \cdot (1 - g_i) + \sum_{i \in N} \Delta_i \cdot (1 - g_i) \right] \quad (10)$$

Similarly we include the optical transponder power consumption at sites without renewable:

$$E_t \cdot \left[ \sum_{m \in N} \sum_{n \in N, m \neq n} w_{mn} \cdot (1 - g_m) \right] \quad (11)$$

We also include the optical amplifier power consumption in the entire network:

$$E_e \cdot \left[ \sum_{m \in N} \sum_{n \in N, m \neq n} f_{mn} \cdot \left\lceil \frac{D_{mn}}{S} \right\rceil \right] \quad (12)$$

The final component is the non-renewable power consumption at nodes with access to renewable energy which is the surplus demand unmet by the solar resource:

$$\sum_{a \in N} g_a \cdot \max \left[ E_r \cdot \left( \sum_{j \in N} C_{aj} + \Delta_a \right) + E_t \cdot \sum_{n \in N} w_{an} - p_a, 0 \right] \quad (13)$$

The objective is to minimize the total non-renewable power consumption in the network and is formulated as:

$$\text{minimize}_{C_{ij}, r_{ij}^{sd}, w_{mn}^{ij}, f_{mn}} \quad (10) + (11) + (12) + (13) \quad (14)$$

The optimization is **subject to** the constraints (3) - (9).

### C. Case 3: Solar with Storage

In the third case ('Solar with Storage') we consider the scenario where a subset of the nodes have access to solar energy as in the second case and in addition have access to energy storage to capture any unused renewable energy. Since the availability of energy storage removes the need to immediately consume renewable energy as it is produced the objective function is to minimize total power consumption.

TABLE I  
SIMULATION PARAMETERS

Parameter	Value
$P_i$	see Table II
$g_m$	$g_2, g_3, g_5, g_{12}, g_{14} = 1$
$B$	40 Gbps
$W$	16
$N$	14 (NSFNET)
$N_m$	Neighbour set derived from topology (Fig. 1)
$\nabla^i$	64
$E_r$	1000 W
$E_t$	73 W
$E_e$	8 W
$S$	80 km
$D_{mn}$	Approximated using straight line distance
$F_{mn}$	5 for all links
$\beta^{max}$	100 kWh (commercial battery)
$t$	1 hour (3600s)

The objective function (1) **minimizes** the total power **subject to** constraints (3) - (9). In addition we account for the energy storage levels. The total power required at a site  $a$  with solar energy (i.e.  $r_a = 1$ ) in the given time period is:

$$P_a^{req} = E_r \cdot \left( \sum_{j \in N, j \neq a} C_{aj} + \Delta_a \right) + E_t \cdot \sum_{n \in N, n \neq a} w_{an} \quad (15)$$

while the amount of power available from renewable energy sources at the same site is the sum of the current solar output and energy storage levels:

$$P_a^{avail} = \frac{\beta_a^{in}}{t} + p_a \quad (16)$$

where  $t$  is the length of the time period in consideration. The storage level at the end of the time period is then calculated as follows:

$$\beta_a^{out} = \max(P_a^{avail} - P_a^{req}, 0) \cdot t \quad (17)$$

An energy storage capacity constraint is also applied:

$$\beta_a^{out} = \min(\beta_a^{out}, \beta_a^{max}) \quad \forall a \in N \text{ where } r_a = 1 \quad (18)$$

## IV. RESULTS

In this section we describe results of our simulation study on the three cases outlined in section III beginning with our simulation settings followed by results on energy and performance. For our test topology we utilize NSFNET spanning four time zones across the continental U.S.A with 14 nodes and 21 bi-directional links depicted in Fig. 1. Each link has up to 5 fibers for activation with 16 wavelengths on each. The capacity of a wavelength is 40 Gbps. We assume a time interval of 1 hour during which traffic demand and renewable energy output remain static and perform the optimizations on an hourly basis. Our study is focused on the operation of the network over a 24 hour period representing a typical day. Table I lists the parameters used in our simulations.

For the traffic model we use the approach outlined in [3]. The average traffic demand between a node pair ranges from 20 Gbps to 120 Gbps depending on time zone and the time of

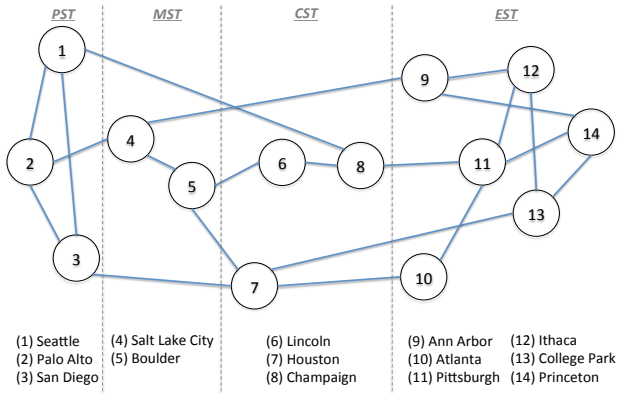


Fig. 1. NSFNET topology

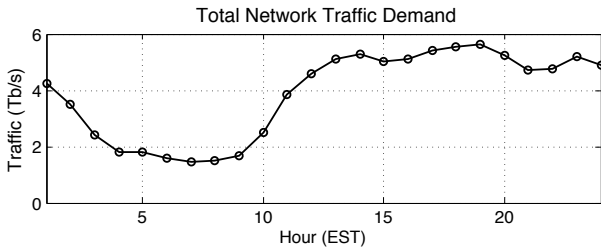


Fig. 2. Total network traffic over 24 hour period

day. Fig. 2 depicts the total system traffic demand. In order to choose sites for solar energy we examined both the average electricity prices and the solar levelized cost of energy (LCOE) in each of the NSFNET site cities found in the National Renewable Energy Laboratory (NREL) database [10]. The LCOE is calculated as the total lifetime system cost over the expected lifetime energy output. It is, therefore, a function of the capital cost and the expected solar irradiation in the city. Currently the average price of electricity is lower than the solar LCOE due to the high capital costs. However, the cost of solar energy systems is decreasing rapidly while electricity prices are expected to rise steadily. For our simulation study we choose the five cities with the smallest difference between average electricity price and solar LCOE which include Palo Alto (6¢/kWh), San Diego (6¢/kWh), Ithaca (9¢/kWh), Boulder (12¢/kWh) and Princeton (13¢/kWh). The average daily solar output for a 200m<sup>2</sup> panel at each of the candidate sites is given in Table II while the aggregate system solar energy availability across the five sites is depicted in Fig. 3.

To solve the convex mixed integer linear programs in cases 1-3 we used CVX, a package for specifying and solving convex programs [11], in conjunction with the Mosek solver for mixed integer linear programs (MILP) [12]. These packages were used with the Matlab software [13].

A. Performance Analysis

As discussed in Section II reconfiguring lightpaths, which includes the removal and addition of lightpaths, is an expensive operation in an IP over WDM network since it disrupts traffic, potentially increases delay and impacts QoS. Fig. 4 compares the number of reconfigurations required in cases 2 and 3. Since

TABLE II  
AVERAGE SOLAR POWER OUTPUT FOR 200 m<sup>2</sup> PANELS

Hour (EST)	Palo Alto (2)	San Diego (3)	Boulder (5)	Ithaca (12)	Princeton (14)
0-6	0 kW	0 kW	0 kW	0 kW	0 kW
6	0 kW	0 kW	0 kW	1 kW	1 kW
8	0 kW	0 kW	4 kW	12 kW	12 kW
10	3 kW	4 kW	12 kW	26 kW	26 kW
12	20 kW	20 kW	26 kW	40 kW	40 kW
14	36 kW	36 kW	40 kW	26 kW	26 kW
16	36 kW	36 kW	36 kW	15 kW	15 kW
18	26 kW	26 kW	26 kW	10 kW	10 kW
20	16 kW	16 kW	16 kW	4 kW	4 kW
22	5 kW	1 kW	4 kW	0 kW	0 kW

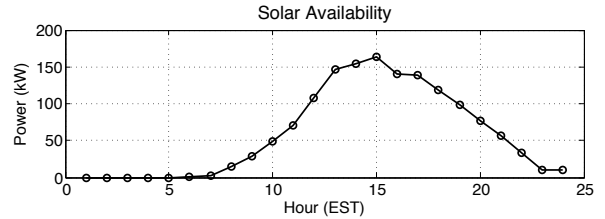


Fig. 3. Solar energy availability across 5 sites

case 1 and case 3 utilize the same objective function they result in the same traffic assignments and hence reconfiguration performance. In case 2, when solar energy becomes available the network requires a high number of reconfigurations in order to re-route traffic through sites with solar. At the peak approximately 4 times as many reconfigurations are required in case 2 when compared to case 3. The addition of energy storage reduces the number of lightpath reconfigurations since traffic does not need to be re-engineered as often. The reconfigurations in case 3 are related to changing traffic demands.

To further study the performance we consider the average path lengths traveled by traffic since this impacts delay in the network. Fig. 5 compares the average traffic path lengths measured in hops. In order to minimize the non-renewable energy as solar energy becomes available the traffic assignments face

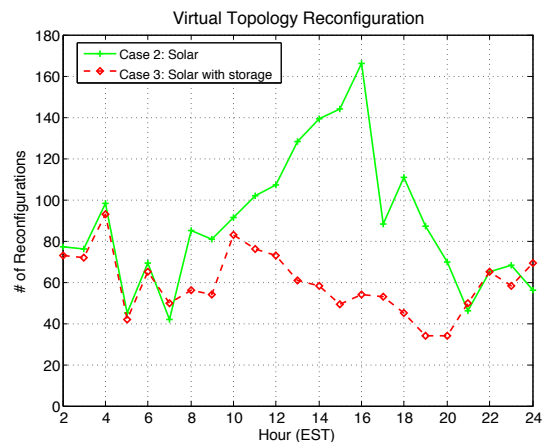


Fig. 4. Total virtual topology lightpath reconfiguration without storage

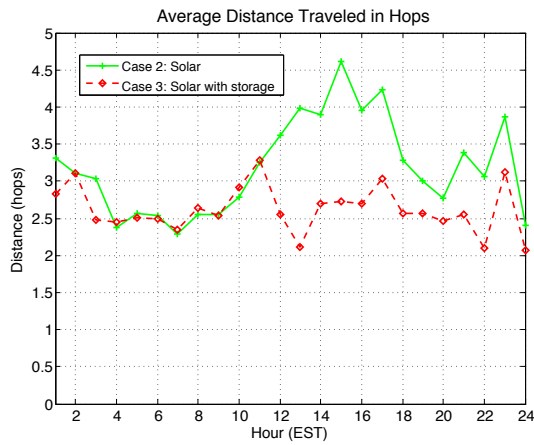


Fig. 5. Average number of physical hops taken by traffic in cases 2 and 3

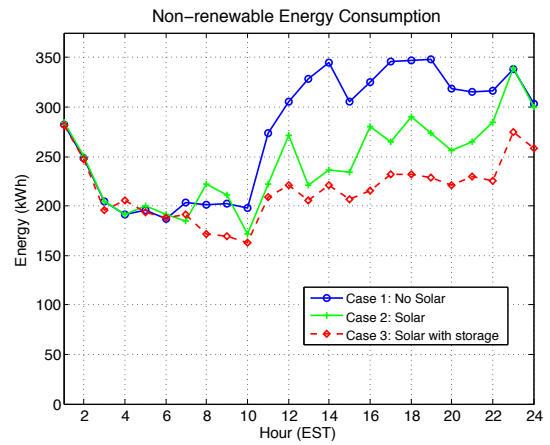


Fig. 7. Hourly consumption of non-renewable energy in cases 1, 2 and 3

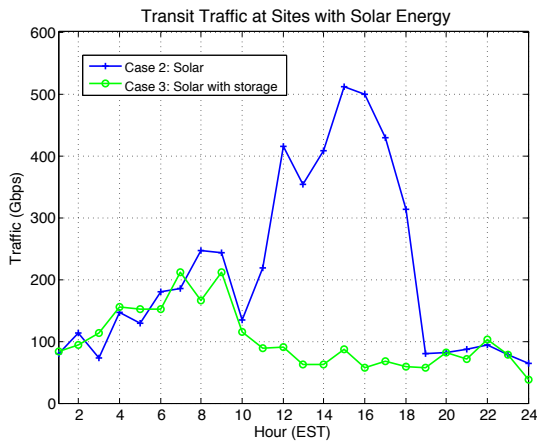


Fig. 6. Transit traffic at sites with solar energy in cases 2 and 3

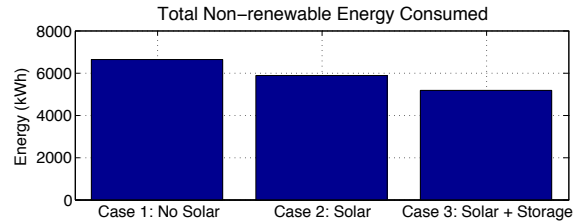


Fig. 8. Total energy supplied by utility over 24 hours (non-renewable)

TABLE III  
ROUTER SIZING IN CASES 2 AND 3

Router Type	Case 2 Ports Required	Case 3 Ports Required
With Solar	34	23
Without Solar	26	23

longer paths.. At times these paths can be twice as long in case 2 as compared to case 3. This can significantly impact delay and, therefore, QoS. The use of energy storage removes the need to engineer traffic to consume energy at specific sites in the network and results in shorter path lengths.

Fig. 6 compares average transit traffic at sites with solar energy in cases 2 and 3. In case 2 the absence of energy storage results in a traffic spike at sites with solar energy during peak energy output hours. With energy storage the spike in transit traffic is not observed. Having a large amount of transit traffic passing through a few sites creates bottlenecks and leaves the network vulnerable to failures. In the first set of results related to the performance of the network we observe that energy storage decouples the availability of renewable energy from the network planning and traffic engineering problem.

### B. Energy Consumption

In this section we study the energy consumption of the three cases. In Fig. 7 the hourly non-renewable energy consumption is compared. Case 1 with no solar energy has the highest non-renewable energy consumption. By adding solar energy in case 2 we witness a reduction in the consumption of non-renewable energy. This is even further reduced in case 3 as energy storage is introduced. Fig. 8 shows the total non-renewable energy

consumption for each case over the 24 hour period. In case 3 the objective function minimizes the total energy consumption. Since unused solar energy is stored and only used as needed, the amount of non-renewable energy in case 3 is the lowest.

The required number of router ports is a crucial factor. Router ports are costly and consume a large amount of power. Tab. III compares the number of router ports required in cases 2 and 3. The difference is more pronounced in routers with solar where the number of ports is reduced from 34 to 23 for the given traffic.

Next we examine how energy storage is being utilized. Fig. 9 shows individual storage levels in case 3. A significant proportion of the generated solar energy is being stored. The peak energy storage levels coincide with the peak output levels. The solar energy production falls off in the evening while the traffic demand remains high. In this period the energy which was stored is consumed by the end of the day.

### C. Right-Sizing Solar Panels

Until now we assumed standardized solar panel sizes and now examine the issue of ideal solar panel sizing. To do this we measure the total power consumption at each site over the course of an average day. We then use these values to calculate the required panel size using the average daily output at each

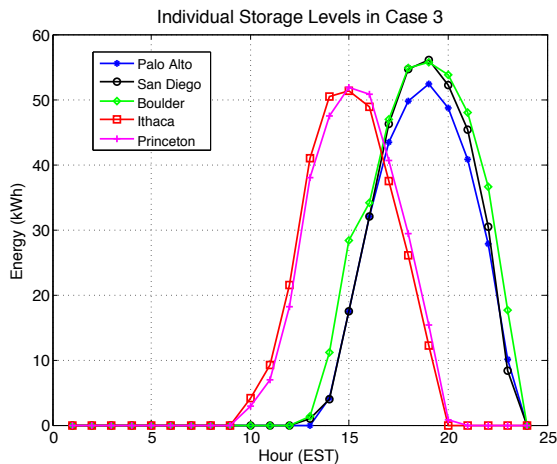


Fig. 9. Energy storage levels

TABLE IV  
SOLAR PANEL RIGHT-SIZING

Site	24hr Energy Consumption	Solar Panel Size
Palo Alto	460 kWh	321m <sup>2</sup>
San Diego	450 kWh	323m <sup>2</sup>
Boulder	481 kWh	300m <sup>2</sup>
Ithaca	453 kWh	338m <sup>2</sup>
Princeton	467 kWh	348m <sup>2</sup>

site so that the router is powered entirely by renewable energy. Table IV summarizes these results. Boulder has the highest energy requirements, likely due to its critical positioning in the topology, but requires the smallest solar panel sizing due to its favorable solar power output. Using these new panel sizes we found that the daily total consumption of non-renewable energy decreased further from 4.5 MW to 4.1 MW. Fig. 10 depicts individual storage levels after increasing the energy storage limit to 200kWh. The storage now provides energy over the 24 hour day to power the entire needs of each router.

### V. DISCUSSION

In this work we examined the integration of renewable energy in IP over WDM networks with dynamic lightpath

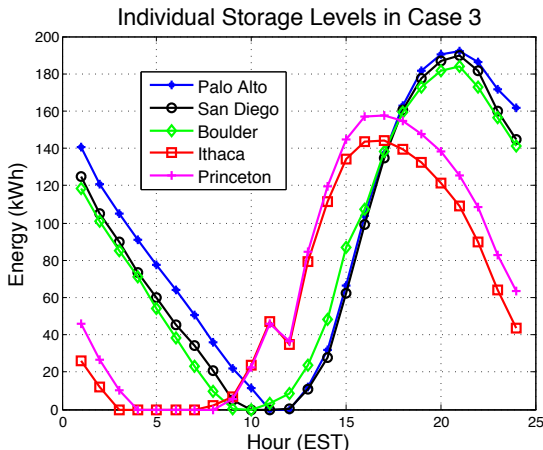


Fig. 10. Total system energy storage levels with right-sized solar panels

assignments. We highlighted the drawbacks of using renewable energy, namely that it necessitates a network planning and traffic engineering strategy which increases delay and complexity in the network. We demonstrated that by adding energy storage in conjunction with renewable energy the energy availability can be decoupled from the operation of the network and further reductions in non-renewable energy consumption could be achieved. As this was a preliminary study we considered average energy and traffic profiles for a typical day. In our future work we aim to study performance with dynamic traffic and weather fluctuations. We also intend to further study the solar panel sizing problem and its impact on the network as well as real-time storage management algorithms which respond to daily/seasonal fluctuations and outages.

Another research direction we aim to pursue is the cost implications for service providers. Renewable energy and energy storage represent additional capital expense. However, there is potential for recovering these costs by exploiting fluctuations in electricity prices, carbon taxes and carbon offsets. In a dynamic pricing environment we can develop algorithms which minimize expenses for a service provider while simultaneously reducing its environmental footprint.

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